Assessment of Packing Density Models and Optimizing Concrete Mixtures

Fayez Moutassem

Ph.D., P.Eng., Department of Civil Engineering and Construction, Bradley University, Peoria, IL., USA

Abstract— *Packing density, an indicator of how efficiently particles fill a certain volume, influences the fresh and hardened properties of concrete. The optimum packing density is conducive to the flowing ability (rheology) and segregation resistance of fresh concrete, whereas the hardened properties and durability improve due to dense particle packing. The maximum packing density is difficult to determine experimentally and therefore, analytical models have been developed for that purpose. This study provides a quantitative assessment of nine packing density models reported in the literature i.e. the applicability of these models for predicting the maximum packing density of aggregates used in concrete applications. Three different compaction methods are considered; loose, rodding and vibration. Results show that only three models are adequate for concrete and that the optimum sand to aggregate ratio occurs between 0.4 and 0.6. Consequently, an improved mixture proportioning methodology that optimizes packing density is developed and proposed.*

Keywords— aggregate packing, concrete aggregates, maximum packing, packing density, void ratio

I. INTRODUCTION

Packing models, which are used to estimate the packing density/voids ratio of solid particles combinations, are tools that can be used to improve the performance of concrete by reducing the free water content and maximizing the solids [1]. The aim is to minimize the porosity of the dry mix which leads to minimizing the binder content. The packing density is defined as:

$$\phi = \frac{V_s}{V_t} = \frac{V_s}{V_s + V_v} = 1 - e \qquad (1)$$

where V_s is the volume of solids, V_t the total volume, V_v the volume of voids, and e the voids ratio.

The packing density of sand and coarse aggregate combinations influences greatly the flowing ability and segregation resistance of concrete [2, 3, 4, 5]. It has been shown that the best workability of concrete for a given cement content and water-to-cement ratio is achieved when the sand-to-aggregate ratio equals to the maximum binary packing of these elements. The relationship between the packing and the rheological properties of fresh concrete, namely the yield stress and plastic viscosity, was studied by Johansen and Andersen [3] among others. In their experiment, the ratio of solids to water content was held constant and the volume fractions of sand and coarse aggregate were varied. Results revealed that the minimum yield stress occurs at sand content between 30% and 40% and that the minimum plastic viscosity at sand content of about 50%. The results also revealed that the minimum yield stress with cement content between 10% and 20% occurs when the coarse aggregate is about 60% by volume. This mixture corresponds to the composition of maximum packing density. These results agree with the reported optimum S/A ratio (by weight) of 0.40 to 0.55 depending on the type and size of aggregate [2, 6].

In addition to improving workability, optimum packing density yields optimum strength, durability, and stability of the hardened concrete by reducing the volume of voids [3, 7, 8, 9]. With the paste volume fixed, the increase in packing density of the aggregate could be employed to increase the strength of the concrete by reducing the water to cement ratio (W/C) while maintaining the same workability [8, 9]. In addition, for the same workability, less paste volume can be used at a fixed W/C, which implies less heat of hydration, less thermal expansion/contraction in the early age and so less drying shrinkage in the long term [9]. Studies has revealed the following; the rheological properties of fresh concrete are function of packing density of concrete [1, 10, 11]; compressive strength is a function of packing density of aggregates [1, 12]. In addition, packing density of aggregates has been shown to be correlate with the packing density of fresh concrete, i.e., given the cement content, increasing the packing density of aggregates has yielded an increase in the packing density of fresh concrete [1].

The current ACI concrete mixture proportioning procedure [13] does not optimize packing density and is crude in its predictions. Optimizing the packing density in design through application of suitable packing density models, would provide the industry with better quality control and economical savings. Since maximum packing density is difficult to measure experimentally, different packing models were developed for dry particles. Therefore, there is a need to evaluate these models for concrete applications and incorporate packing density in current mixture proportioning. The objectives of this research include, 1) evaluating the applicability of the many particle packing density models reported in the literature for predicting the maximum packing density of aggregates used in concrete applications and, 2) developing a mixture proportioning methodology that incorporates and optimizes packing density of aggregates.

II. EXPERIMENTAL AND ANALYTICAL INVESTIGATION

2.1 Packing Density Models

Nine models, published in the literature for calculating the packing density of dry mixture were evaluated. Table 1 provides a list of the proposed equations for calculating the packing density.

Table 1. Published Aggregate Packing Density Models

Model	Equation
FM	$\phi = \left(\frac{y_1}{\phi_1} + y_2\right)^{-1}$ for $y_1 >> y_2$ & $\phi = \frac{\phi_2}{y_2}$ for $y_1 << y_2$
AGM	$\phi = \frac{\phi_2}{1-y_1}$ for $y_1 < y^*$ & $\phi = \left(\frac{y_1}{\phi_1} + (1-y_1)\left(1+0.9\frac{d_1}{d_2}\right)\right)^{-1}$ for $y_1 > y^*$; $y^* = \frac{\frac{\phi_1}{\phi_2} - \left(1+0.9\frac{d_1}{d_2}\right)\phi_1}{1+\frac{\phi_1}{\phi_2} - \left(1+0.9\frac{d_1}{d_2}\right)\phi_1}$
MTM	$\phi = \left(\frac{y_1}{\phi_1} + \frac{y_2}{\phi_2} - y_2\left(\frac{1}{\phi_2} - 1\right)k_d k_s\right)^{-1}$
LPM	$\phi = \frac{\phi_1}{1-y_2 f(r)}$ for $y_1 >> y_2$ & $\phi = \frac{\phi_2}{1-(1-\phi_2)y_1 g(r)}$ for $y_1 << y_2$
MLPM	$\phi = \left(\sum_{j=1}^{i-1}\left[V_j - (V_j - 1)g(r)\right]X_j + V_i X_i + \sum_{j=i+1}^{n} V_j\left[1-f(r)\right]X_j\right)^{-1}$
MPM	$\phi = \left(\sum_{i=1}^{n} X_i V_i + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\beta_{ij}X_i X_j + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\gamma_{ij}X_i X_j (X_i - X_j)\right)^{-1}$
LMPM	$\phi = V_i^{mix}\sum_{j=M}^{N} X_j + V_{unmxg}^{S} + V_{unmxg}^{L}$
TPM	$\phi = \left(\frac{U_0^{"}}{(1+U_1^{"}+U_0^{"})}U_1^{"} + 1\right)^{-1}$
CPM	$K = \sum_{i=1}^{n}\frac{\frac{y_i}{\beta_i}}{\frac{1}{\phi} - \frac{1}{\gamma_i}}$

ϕ_i = packing density of particle i; y_i = volume fraction of particle i; d_i = diameter of particle i; k_d = factor that determines the influence of the diameter ratio; k_s = statistical factor; f(r) & g(r) = interaction functions between components i & j; V_i &V_j = partial specific volumes of particles i & j, respectively; X_i&X_j = volume fractions of particles i & j, respectively; β_{ij}&γ_{ij} = statistical coefficients dependent on the size ratio and initial porosity; V_i^{mix} = partial specific volumes of the controlling mixture; V^{s}_{unmxg} = partial specific volumes of the small particles of un-mixing effect; V^{L}_{unmxg} = partial specific volumes of the large particles of mixing effect; $U^{”}_{0}$ & $U^{”}_{1}$ = value on the voids ratio diagram of CA and FA points, respectively; K = partial compaction index that depends on the method of compaction; β_i = residual packing density of particle i; γ_i = virtual packing density of component i in the mixture.

They include the Furnas Model, FM [15]; Aim and Goff Model, AGM [16]; Modified Toufar Model, MTM [17]; Linear Packing Model, LPM [18]; Modified Linear Packing Model, MLPM [19]; Mixture Packing Model, MPM [20]; Linear-Mixture Packing Model, LMPM [20]; Theory of Particle Mixtures, TPM [21]; and Compressible Packing Model, CPM [1]. Most of the packing models developed, excluding TPM and CPM, are based on the assumption that particles are monosized and spherical in shape. This limitation was overcome by introducing a characteristic diameter for the aggregates, and for some by grouping the aggregate according to specified size and measuring their packing density separately. The characteristic diameter can be determined using the Rosin-Raimmler-Sperling-Benett distribution [22].

2.2 Experimental Program

An experimental program was developed to evaluate the suitability of the nine proposed models for calculating the packing density of dry particles. The particles used were crushed coarse aggregate (CA) with 0.75 in. (19 mm) maximum size and an average specific gravity of 2.75, and fine aggregate (FA) with an average fineness modulus of 2.69 and an average specific gravity of 2.69. These properties were obtained in accordance with ASTM C127 [23] and ASTM C128 [24] for CA and FA, respectively. The particle size distributions of CA and FA were found to conform to the ASTM C136/C136M [25] specification requirements, as shown in Fig. 1. The characteristic diameters for CA and FA are 0.49 in. (12.4 mm) and 0.045 in. (1.15 mm), respectively. The mean diameters for CA and FA, corresponding to 50% retained, are 0.38 in. (9.61 mm) and 0.028 in. (0.72 mm), respectively.

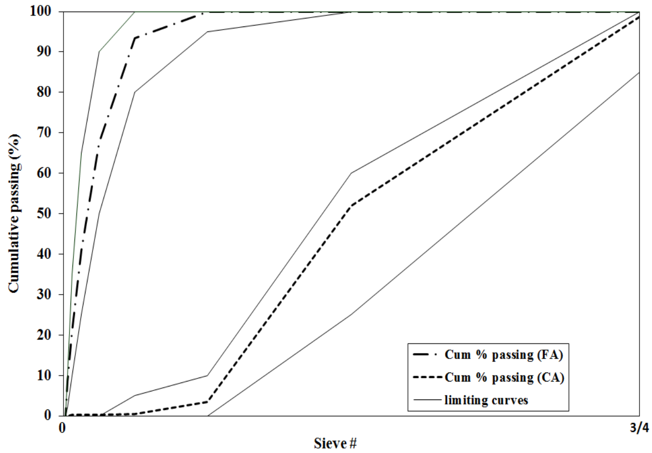

Fig. 1: Particles Size Distribution for Aggregates

The packing density of dry aggregates also depends on the method of compaction. Stovall et al. [18] recommended using the vibrated plus pressure packing density for the CPM, Johansen and Anderson [3] suggested the rodded packing density, and Dewar [21] proposed using the loose packing density for the TPM. To study the influence of the method of compaction on the packing density, three different methods of compaction were used: loose, rodding, and vibration. The packing density of oven dried fine and coarse aggregates was measured using cylindrical steel containers whose height and diameter is greater than 5 times the maximum aggregate size. The packing density defined as the volume of solids per unit volume, is calculated according to

$$\alpha = \frac{V_s}{V_c} = \frac{W}{V_c \gamma_s} \quad (2)$$

where W is the weight of the aggregate, γ_s the specific gravity, V_s the volume of solid and V_c the volume of the container.

The procedure followed for measuring the packing density is as follows:

Loose Packing

1. Fill container with aggregate by discharging it from a height 50 mm above the top.
2. Remove surplus aggregate and finish the surface by rolling a steel rod across the top.

Rod Packing

1. Fill the container in 3 layers with each layer compacted 25 times with a rod according to ASTM C29/C29M [26].
2. Remove surplus aggregate and finish the surface by rolling a steel rod across the top.

Vibration Packing

1. Fill the container with aggregate and do not finish the surface, i.e. leave the surplus aggregate.
2. Place the container on a vibrating table and vibrate for a period of 2 minutes.
3. Remove surplus aggregate and finish the surface by rolling a steel rod across the top.

III. COMPARISON OF PREDICTED AND EXPERIMENTAL RESULTS

3.1 Results and Analysis

Table 2 gives the measured bulk densities and grain densities corresponding to the three methods of compaction. The distribution of packing density as a function of volume fraction for different compaction methods is shown in Fig. 2. Volume fraction is defined as the ratio of fine aggregates, corresponding to sand, divided by the total aggregates, corresponding to sand and coarse aggregates. The results of Fig. 2 show that the vibration method yields the highest packing density whereas loose compaction yields the lowest values. They also reveal that the maximum packing density occurs when the volume fraction is approximately 0.5. One also observes that the packing density of the FA is greater than that of the CA. These results are consistent with those reported in the literature [2, 3, 4, 5, 6]. Closer examination of the results indicates that the coefficient of variance is higher for the loose packing in comparison to the other two methods of compaction, and for the volume fraction between 0.0 and 0.5 regardless of the method of compaction. For the CPM model, the packing density for every sieve size needs to be measured for FA and CA. The results are summarized in Table 3.

Table 2. Measured Bulk Densities

Volume fraction	Bulk densities ± Standard deviation, lb/ft^3 (kg/m^3)			Density, lb/ft^3 (kg/m^3)
	Loose	Rodding	Vibration	
0.0	87.5±1.19 (1401±19)	94.6±0.12 (1515±2)	97.0±0.44 (1554±7)	171.7 (2750)
0.1	93.8±1.44 (1503±23)	101.6±0.56 (1628±9)	103.7±0.19 (1661±3)	171.1 (2740)
0.2	99.7±0.50 (1597±8)	108.4±0.31 (1736±5)	110.2±1.56 (1765±25)	171.1 (2740)
0.3	104.1±1.81 (1668±29)	114.6±0.44 (1835±7)	117.1±1.06 (1875±17)	170.4 (2730)
0.4	110±2.18 (1762±35)	119.4±0.75 (1913±12)	121.5±0.94 (1947±15)	170.4 (2730)
0.5	113.2±0.56 (1813±9)	121.4±2.12 (1945±34)	124.5±0.69 (1995±11)	169.8 (2720)
0.6	112.7±1.06 (1806±17)	120.4±0.06 (1929±1)	124.0±0.06 (1987±1)	169.2 (2710)
0.7	110.2±0.25 (1765±4)	117.7±0.44 (1886±7)	121.4±0.44 (1945±7)	169.2 (2710)
0.8	107.7±0.25 (1725±4)	115.0±0.25 (1842±4)	119.3±0.12 (1911±2)	168.6 (2700)
0.9	104.7±0.81 (1677±13)	110.9±0.50 (1777±8)	116.2±0.19 (1862±3)	168.6 (2700)
1.0	98.7±0.87 (1581±14)	106.5±1.0 (1706±16)	113.8±0.06 (1823±1)	167.9 (2690)

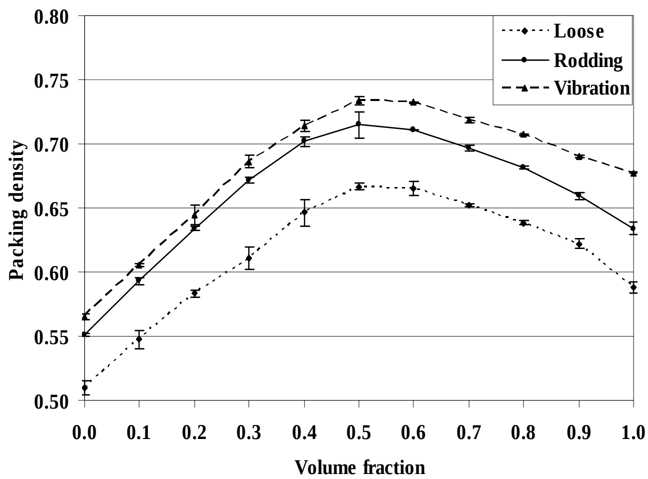

Fig. 2: Measured Packing Density vs. Volume Fraction

Table 3. Experimental Packing Densities

Sieve Size, in. (mm)	Packing densities ± Standard deviation		
	Loose	Rodding	Vibration
0.75 (19)	0.463±0.0065	0.501±0.0047	0.519±0.0043
0.375 (9.5)	0.463±0.0065	0.501±0.0047	0.519±0.0043
0.187 (4.75)	0.460±0.0037	0.498±0.0003	0.516±0.0065
0.0937 (2.36)	0.461±0.0009	0.490±0.0015	0.505±0.0009
0.0469 (1.18)	0.462±0.0015	0.488±0.0015	0.514±0.0000
0.0234 (0.60)	0.451±0.0017	0.487±0.0009	0.508±0.0052
0.0117 (0.30)	0.445±0.0009	0.483±0.0015	0.505±0.0015
0.0059 (0.15)	0.440±0.0015	0.486±0.0015	0.507±0.0015
0.0030 (0.075)	0.437±0.0009	0.494±0.0000	0.535±0.0009

Packing density as a function of volume fraction obtained using the nine packing density models for the three methods of compaction is shown in Fig. 3 to 5, respectively.

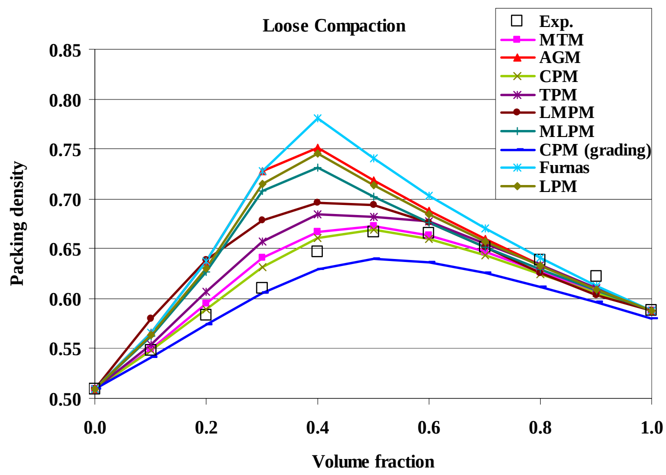

Fig. 3: Models Evaluation- Loose Compaction

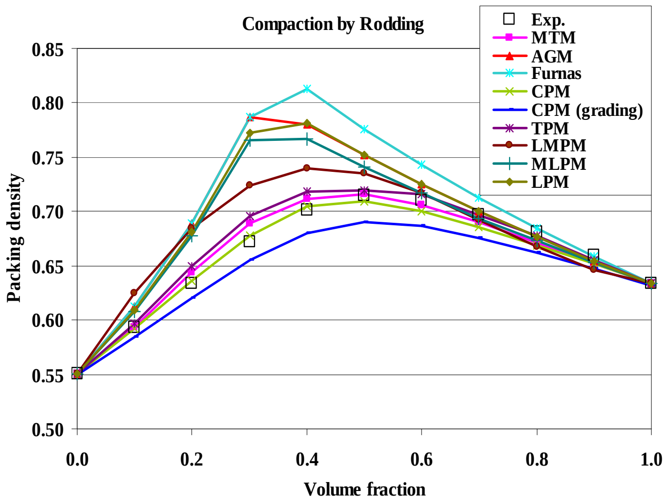

Fig. 4: Models Evaluation- Rodding

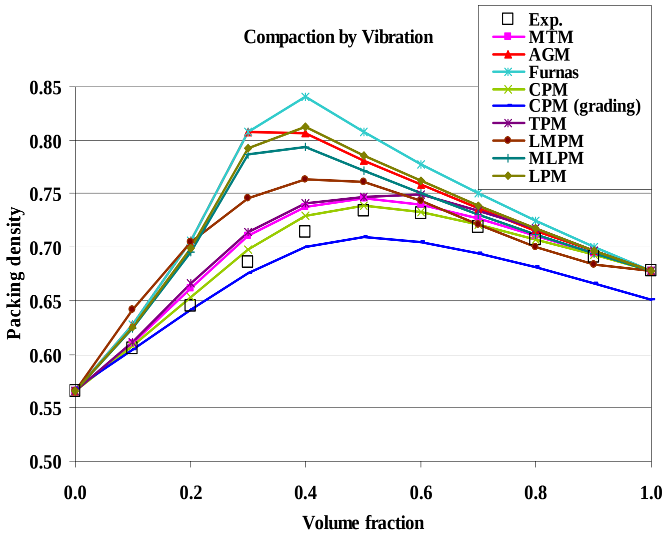

Fig. 5: Models Evaluation- Vibration

Evaluation of the packing density was done two times using the CPM. The first method referred to as CPM grading employs the experimental packing density results obtained for every sieve size, and the method noted as CPM uses the characteristic diameter as well as the experimental data obtained for FA and CA. Results in Fig. 3 reveal that all nine models with the exception of CPM (grading) have overestimated the packing density when the volume fraction is below 0.5 and underestimates when the volume fraction is close to 1.0 for loose compaction. Similar observation can be made from the results shown in Fig. 4 and Fig. 5, corresponding to compaction by rodding and by vibration, respectively. Furthermore, the CPM (grading) is consistently underestimating the packing density for the whole volume fraction range and for the three compaction methods. In the range of interest, corresponding to volume fraction between 0.4 and 0.6, for all three methods of compaction, the CPM, MTM, and TPM gave comparative results whereas, Furnas, AGM, LPM, MLPM, and CPM (grading) gave poor results.

The adequacy of the models predictions was assessed by calculating the percent difference errors between the measured and calculated packing densities corresponding to volume fractions 0.3 to 0.7. The maximum values obtained for the loose, rodding, and vibration method of compaction are given in Table 4 and in ascending order. These results indicate that the CPM has the lowest percent difference errors followed by the MTM, CPM (grading), TPM, LMPM, MLPM, LPM, AGM, and Furnas.

Table 4. Measured vs. Calculated Packing Densities

Packing Models	Percent difference (%)		
	Loose	Rodding	Vibration
CPM_{error}	3.4	1.6	2.1
MTM_{error}	4.9	2.5	3.5
CPM(grading) $_{error}$	4.3	3.5	3.7
TPM_{error}	7.7	3.6	4.0
$LMPM_{error}$	11.1	7.8	8.7
$MLPM_{error}$	16.0	14.0	14.6
LPM_{error}	17.0	15.0	15.5
AGM_{error}	19.2	17.2	17.6
$Furnas_{error}$	20.8	17.2	17.6

For concrete aggregates, the CPM using the characteristic diameter is found to give the best results and requires less computational work in comparison to the CPM (grading). Moreover, the MTM and TPM are also found adequate for predicting the packing density. When comparing the results obtained for the three methods of compaction, one observes that the packing density obtained using the loose compaction method possesses the largest percent difference error in comparison to rodding and vibration. These results are expected as it is less stringent in comparison to the other two methods.

3.2 Discussion of Results

The Furnas and AGM models assume that the amount of fine particles is either much less than the amount of coarse particles to fill the voids or much higher so as to act as a media in which the coarse particles are embedded. These models are expected to yield better estimates when the volume fraction of fine particles is either too large or too small. In addition, these models are more applicable to small diameter ratios. Therefore, they are not suitable to calculate the packing density of aggregates used in concrete.

The MTM assumes that diameter ratios greater than 0.22 will be too large to be situated within the interstices between the larger particles. The result is a packing of the matrix that may be considered as a mixture of packed areas mainly consisting of larger particles, and packed areas that mainly consist of smaller particles with larger particles distributed discretely throughout the matrix of smaller particles. Therefore, this model is expected to yield good estimates for large diameter ratios. The results show that the model still gives very good results for small diameter ratios. The AGM and the MTM assume that fine and coarse aggregate are of different sizes. This assumption may lead to problems when the two aggregates have overlapping fractions, i.e. when a substantial part of the fine aggregates is of the same size as a substantial part of the coarse aggregates, and when their characteristic diameters are fairly different. However, for the CA and FA used in concrete, most of the fractions do not overlap and so this problem can be considered negligible. As a result, the MTM is found to give good estimates of the packing density of aggregate used in concrete.

The linear packing model considers the unmixing mechanism because it always assumes that there is only one controlling component. Therefore, the predictions can be adequate when the size ratios are large. The model results are found poor even for small diameter ratios. On the other hand, the mixture packing model only considers the mixing mechanism and cannot satisfactorily account for the packing formed by the unmixing mechanism. Therefore, the resultant predictions may not match the measurements well if the size ratios involved are small. The linear-mixture packing model is a combination of both models since it considers both the unmixing and mixing mechanisms and is therefore superior to both models. However, it fails to accurately predict the porosity when there is a large difference in initial porosity. In other words, they can only be used for packing systems in which the maximum difference between initial porosity is relatively small, less than 0.1. The difference between the initial porosities of FA and CA is a little higher than 0.1 which explains why the results are not as accurate as expected.

In the theory of particle mixtures, each real particle in a mixture of single-sized particles is assumed to be associated with a single corresponding void where in reality, each solid particle shares voids with other particles. This model is based on the sequence of combining the finest materials before adding the coarsest material. It assumes that a graded material can be represented by a single sized material (characteristic diameter) having the same void ratio as the graded material. This would create problems if a material with a gap in its grading is to be combined with a material with a mean size within that gap. This is usually not a problem for concrete and it was shown to produce good estimates of the packing density.

The CPM is a refined version of the linear packing model where the virtual packing and compaction index concepts are introduced and the wall and loosening effects are accounted for [1]. The CPM covers combinations of any number of individual aggregate fractions, having any type of size distribution. Initially, the model was applied by considering every particle retained on every sieve size separately and by measuring its packing density, CPM (grading). However, it was determined that this approach is not needed and that the CPM can be applied with high accuracy by determining the characteristic diameters of the FA and CA and measuring the packing density of each. This research has shown that applying the CPM using characteristic diameters has resulted in accurate predictions. Results of this study has shown that the CPM, MTM, and TPM models can all be applied to predict the maximum packing density of aggregates used in concrete and can therefore be used to assess the flowability, segregation resistance, hardened properties and durability of concrete on the basis of their aggregates' geometry and composition.

IV. NEW METHODOLOGY FOR DESIGN OF CONCRETE MIXTURES

The current ACI 211.1 [13] concrete mixture proportioning procedure does not account for diversities in aggregate nature and their packing densities and is therefore crude in its predictions [14]. Results of the evaluation of packing models have shown that only three models (CPM, MTM, and TPM) can adequately predict the packing density of aggregates used in concrete. On this basis, an improved ACI mixture proportioning methodology that incorporates and optimizes packing density is proposed as follows:

1) From $f'_{c\ required}$ and according to ACI 211.1 guidelines, estimate w/c
2) From required slump according to ACI 211.1 guidelines, estimate water content
3) From w/c and water content, calculate the cement content.
4) Calculate the total volume of aggregates from $V_{agg}=1-V_w-V_c-V_{air}$
5) Given the gradations of fine and coarse aggregates, determine the optimum volume fraction of fine aggregate to total volume of aggregate (V_{FA}/V_{agg}) via optimization of ϕ^*_{agg} using any of the 3 packing models (CPM, MTM, or TPM).
6) From optimum V_{FA}/V_{agg} determined in step 6 and from V_{agg} calculated in step 5, Compute the volume of fine aggregates V_{FA}.
7) Calculate the volume of coarse aggregates from $V_{CA} = V_{agg} - V_{FA}$

8) Knowing the volumes and properties of the materials, calculate the weights.

V. SUMMARY AND CONCLUSIONS

This study provided a quantitative assessment of applicability of nine particle packing density models reported in the literature for predicting the maximum packing density of aggregates used in concrete applications using three different compaction methods. Consequently, a modified version of ACI 211.1 mixture proportioning methodology that incorporates and optimizes packing density of aggregates was proposed. The following conclusions are drawn from the packing density models investigation:

1. Of the nine packing density models evaluated, only the CPM, MTM, and TPM are found to correctly predict the packing density of aggregate used in concrete.
2. Regardless of the method of compaction, the optimum packing density occurs when the volume fraction of fine aggregate is between 0.4 and 0.6, consistent with the ACI 211.1 [13] guidelines.
3. Three models, CPM, MTM, and TPM, can be used for evaluating concrete mixes and for improving the properties of concrete.

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