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Kantorovich-Vlasov Method for Simply Supported Rectangular Plates under Uniformly Distributed Transverse Loads

Nwoji C.U., Mama B.O., Onah H.N., Ike C.C.

Dept of Civil Engineering, University of Nigeria, Nsukka, Enugu State, Nigeria

Abstract— *In this study, the Kantorovich-Vlasov method has been applied to the flexural analysis of simply supported Kirchhoff plates under transverse uniformly distributed load on the entire plate domain. Vlasov method was used to construct the coordinate functions in the x direction and the Kantorovich method was used to consider the assumed displacement field over the plate. The total potential energy functional and the corresponding Euler-Lagrange equations were obtained. This was solved subject to the boundary conditions to obtain the displacement field over the plate. Bending moments were then obtained using the moment curvature equations. The solutions obtained were rapidly convergent series for deflection, and bending moments. Maximum deflection and maximum bending moments occurred at the center and were also obtained as rapidly convergent series. The series were computed for varying plate aspect ratios. The results were identical with Levy-Nadai solutions for the same problem.*

Keywords— Kantorovich-Vlasov method, Levy-Nadai method, convergent series, total potential energy functional, Euler-Lagrange differential equation.

I. INTRODUCTION

Plates are widely used in many engineering structures like roof and floor slabs, bridge deck slabs, foundation footings, bulkheads, watertanks, turbine disks, spacecraft panels and ship hulls. Plates may be subjected to static flexural loads, dynamic loads or inplane loads, resulting in three types of analysis, namely – static flexural analysis, dynamic flexural analysis and stability analysis [1]. The deformation of plates is usually defined as the deformation of the middle surface, which is a plane surface equidistant from the top and bottom faces of the plate [2]. The thickness of the plate *h*, is the distance between the top and bottom faces, measured in a direction normal to the middle plane.

Generally, plates are classified into three groups: thin plates, moderately thick plates and thick plates, depending upon the thickness of the plate [2]. Plates are also classified depending on the nature of deformation and material properties as plates with small deformations, plates with large deformations, orthotropic plates, isotropic plates and anisotropic plates. They can also be classified according to their shapes as rectangular, circular, triangular, skew and elliptical plates.

Thin plates are plates for which the thickness is very small when compared with the smaller of the in plane dimensions. For most practical applications, thin plates have a ratio of thickness to the smaller in plane dimension less than 0.05 [2]. The plate deformation is considered small if the maximum deformation is less than or equal to its thickness.

The most widely used plate theory is the Kirchhoff-Love plate theory, also called the classical thin plate theory [3]. The Kirchhoff-Love plate theory is based on the Kirchhoff-Love hypothesis which makes assumptions similar to those in the Bernoulli-Navier hypothesis used in the theory of thin (shallow) beams.

For large deflections, the flexure of the plate is accompanied with inplane stresses which depend not solely on the deflection but also on the type of edge supports [2]. The incorporation of inplane stresses in the behaviour of plates results in non-linear differential equations which present theoretically rigorous demands for solutions.

For a good number of engineering applications, the Kirchoff-Love theory gives sufficiently accurate results, the accuracy however reduces with increased plate thickness, with localized loads, and in problems of stress concentrations around openings [2], [4]. Again, the Kirchhoff-Love plate theory sometimes uses an approximate incomplete set of boundary conditions which results in errors, especially near the boundary regions as well as in reactions [2], [4].

Thick plate problems can only be formulated and solved using the three dimensional theory of elasticity. For many engineering problems, solutions based on the three dimensional theory of elasticity present mathematically rigorous demands for solution, and thus far, only simple

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plate problems have been solved using this theory [2], [4], [5], [6], [7].

Several other theories have been proposed to describe the behaviour of plates under loads. They include – Reissner's sixth order plate theory [8], [9] for moderately thin plates; Von Karman plate theory for plates under large deformation; Mindlin plate theory for moderately thick plates, shear deformation plate theories [10, 11, 12].

The focus of this paper is the Kirchhoff plate theory. Several methods have been used to analyse plates. They are broadly classified as analytical methods and numerical methods. Analytical methods include Navier and Levy methods [13]. Numerical methods which seek approximate solutions to the plate problem include: variational methods, finite element and finite difference methods and finite strip methods [14, 15, 16].

Kirchhoff-Love hypothesis

The Kirchhoff-Love hypothesis for the small deflection theory of thin homogeneous elastic plates are [2]:

i. the deflection of the middle surface is small compared to the thickness
ii. the middle surface is unstrained subsequent to bending, and hence is a neutral surface
iii. plane sections initially normal to the middle surface remain plane and normal to the middle surface even after deformation
iv. the stress (σ_z) normal to the middle surface is very small compared to the other two normal stress components $(\sigma_{xx} \text{ and } \sigma_{yy})$ and hence σ_z can be neglected.

These assumptions reduce the three dimensional elasticity problems to two dimensions.

II. RESEARCH AIM AND OBJECTIVES

The aim of this study is to apply the Kantorovich-Vlasov method to the analysis of simply supported thin rectangular plates under uniformly distributed transverse loads. The specific objectives are:

i. to use the Vlasov procedure to derive suitable coordinate shape functions for the simply supported plate in the *x* coordinate direction.
ii. to obtain the total potential energy functional Π for the plate bending problem based on the Vlasov shape functions derived.
iii. to obtain the Euler Lagrange differential equations for the total potential energy functional derived.
iv. to solve the Euler-Lagrange differential equations subject to the boundary conditions of the plate in the *y* coordinate direction.
v. to use the bending moment displacement relations and obtain the bending moment expression.
vi. to obtain the maximum values of deflection and bending moments at the centre of the plate for various aspect ratios.

III. THEORETICAL FRAMEWORK

The strain energy of a plate for linear elastic behaviour is given by the volume integral:

$$U = \frac{1}{2}\iint_R \int_{-h/2}^{h/2} \sigma_{ij}\,\varepsilon_{ij}\; dz\,dx\,dy \qquad (1)$$

where σ_{ij} are stresses, ε_{ij} are strains, *h* is the plate thickness, *R* is the two dimensional domain of the plate on the *xy* plane.

From the stress-strain relations, the strain energy expression can be expressed in terms of strains and stresses as follows:

$$U = \frac{E}{2(1-\mu^2)}\iint_R \int_{-h/2}^{h/2} \left[\varepsilon_{xx}^2 + 2\mu\varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{yy}^2 + 2(1-\mu)\varepsilon_{xy}^2\right] dz\,dx\,dy \qquad (2)$$

or

$$U = \frac{1}{2E}\iiint_v \Big\{\Big(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\mu(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z\Big)$$

$$+ 2(1-\mu)(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)\Big\}\,dx\,dy\,dz \qquad (3)$$

where $\varepsilon_{xx}, \varepsilon_{yy}$ are normal strains, σ_{xx}, σ_{yy} are normal stresses, ε_{xy} is the shear strain, σ_{xy} is the shear stress, μ is the Poisson's ratio, *E* is the Young's modulus of elasticity, and *V* is the three dimensional region of the plate.

If the distributed transverse loads *p*(*x*, *y*) are assumed to act on the middle surface of the plate and *w*(*x*, *y*) is the vertical displacement function of the plate middle surface, the potential energy of the external distributed loads is given by:

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$$V_e = -\iint_R p(x,y)\, w(x,y)\, dxdy \quad (4)$$

For small deformations, the strain-displacement relations are:

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2} \quad (5)$$

$$\varepsilon_{yy} = -z\frac{\partial^2 w}{\partial y^2} \quad (6)$$

$$\varepsilon_{xy} = -z\frac{\partial^2 w}{\partial x \partial y} \quad (7)$$

where $w(x, y)$ is the transverse deflection.
Using the stress-strain laws,

$$\sigma_{xx} = -\frac{Ez}{(1-\mu^2)}\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) \quad (8)$$

$$\sigma_{yy} = -\frac{Ez}{(1-\mu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) \quad (9)$$

$$\tau_{xy} = -\frac{Ez}{(1-\mu^2)}\frac{\partial^2 w}{\partial x \partial y} \quad (10)$$

$$\tau_{zx} = \tau_{zy} = \sigma_{zz} = 0 \quad (11)$$

where E is the Young's modulus of elasticity, μ is the Poisson's ratio, τ_{zx}, τ_{zy} and τ_{xy} are shear stresses, σ_{zz} is normal stress.
The total potential energy functional becomes:

$$\Pi = U + V_e$$

$$\Pi = \frac{D}{2}\iint_R \left[(\nabla^2 w)^2 + 2(1-\mu)\left(\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}\right)\right)\right] dxdy - \iint_R pw\, dxdy \quad (12)$$

where $D = \frac{Eh^3}{12(1-\mu^2)}$ (13)

D is the bending rigidity, h is the plate's thickness.

IV. APPLICATION OF THE KANTOROVICH-VLASOV METHOD TO THE BENDING ANALYSIS OF SIMPLY SUPPORTED KIRCHHOFF PLATES

Consider the rectangular Kirchhoff-Love plate shown in Figure 1.

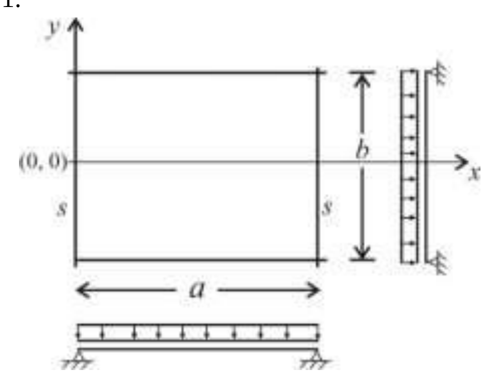

Fig.1: Rectangular Kirchhoff-Love plate under uniformly distributed load.

The coordinate axis is chosen as shown in Figure 1, in order to take advantage of the symmetry of the problem in the y coordinate direction. The plate is subject to a transverse distributed load of intensity q over the whole plate domain which is given by $0 \le x \le a,\ -b/2 \le y \le b/2$

4.1 Vlasov procedure for finding the displacement shape function in the *x*-direction

Following Vlasov procedure, the displacement shape function in the x-direction are the eigenfunctions of an Euler-Bernoulli beam on the x-axis. The differential equation of free vibration of Euler-Bernoulli beams is

$$EI\frac{d^4 X(x,t)}{dx^4} + \bar{m}\frac{d^2 X(x,t)}{dt^2} = 0 \quad (14)$$

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where EI = flexural rigidity of beam, $\bar{m}$ = mass per unit length of beam, $X(x, t)$ are displacement functions in the x direction. For harmonic vibrations, X will vary with space, x and time, t variables as:

$$X(x,t) = f(x)\sin\omega_n t$$

where ω_n is the natural frequency, and $f(x)$ is the displacement shape function in the x-direction.

Then the differential equation of free vibration becomes:

$$\left(f^{iv}(x) - \frac{\bar{m}\omega_n^2}{EI} f(x)\right)\sin\omega t = 0 \qquad (15)$$

or $$f^{iv}(x) - \frac{\bar{m}\omega_n^2}{EI} f(x) = 0 \qquad (16)$$

where $f^{iv}(x) = \dfrac{d^4 f(x)}{dx^4}$

or $$f^{iv}(x) - \frac{\beta^4}{a^4} f(x) = 0 \qquad (17)$$

where $\dfrac{\bar{m}\omega_n^2}{EI} = \dfrac{\beta^4}{a^4}$

The general solution of Equation (17) is:

$$f(x) = c_1 \sin\frac{\beta x}{a} + c_2 \cos\frac{\beta x}{a} + c_3 \sinh\frac{\beta x}{a} + c_4 \cosh\frac{\beta x}{a} \qquad (18)$$

where c_1, c_2, c_3 and c_4 are integration constants, which can be found from the boundary conditions, and β is the root of the characteristic equation.

The eigenfunctions f_m for the mth mode of vibration are obtained as

$$f_m(x,t) = \left(c_{1m} \sin\frac{\beta_m x}{a} + c_{2m} \cos\frac{\beta_m x}{a} + c_{3m} \sinh\frac{\beta_m x}{a} + c_{4m} \cosh\frac{\beta_m x}{a}\right) \qquad (19)$$

For simple supports at $x = 0$, $x = a$ the boundary conditions are

$$f_m(0) = f_m''(0) = 0$$
$$f_m(a) = f_m''(a) = 0 \qquad (20)$$

Using the boundary conditions in Equation (20), the following algebraic homogeneous equation is obtained:

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ \sin\beta_m & \cos\beta_m & \sinh\beta_m & \cosh\beta_m \\ -\sin\beta_m & -\cos\beta_m & \sinh\beta_m & \cosh\beta_m \end{pmatrix} \begin{pmatrix} c_{1m} \\ c_{2m} \\ c_{3m} \\ c_{4m} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (21)$$

The solution yields

$$f_m(x) = c_{1m} \sin\frac{m\pi x}{a} \qquad m = 1, 2, 3, \ldots \qquad (22)$$

Thus, the eigenfunction is obtained as

$$f_m(x) = \sin\frac{m\pi x}{a} \qquad (23)$$

where $m = 1, 2, 3, 4, \ldots$

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4.2 Kantorovich procedure to obtain the total potential energy functional

Following Kantorovich procedure, the plate deflection is considered as

$$w(x,y)=\sum_{m=1}^{\infty} g(y)\sin\frac{m\pi x}{a} \quad (24)$$

where $g(y)$ is an unknown function of the independent coordinate variable y. Substitution of Equation (24) into the total potential energy functional yields after simplification:

$$\Pi=\frac{D}{2}\sum\int_{-b/2}^{b/2}\left((g''(y))^2-2\left(\frac{m\pi}{a}\right)^2 g(y)\,g''(y)+\left(\frac{m\pi}{a}\right)^4(g(y))^2\right)dy\int_0^a \sin^2\frac{m\pi x}{a}dx$$
$$+\int_{-b/2}^{b/2} qg(y)dy\int_0^a \sin\frac{m\pi x}{a}dy \quad (25)$$

But

$$\int_0^a \sin\frac{m\pi x}{a}dx=\frac{a}{m\pi}(1-\cos m\pi)=\frac{2a}{m\pi} \quad (26)$$

where m = odd = 1, 3, 5, …

and $$\int_0^a \sin^2\frac{m\pi x}{a}dx=\frac{a}{2} \quad (24)$$

$$\Pi=\sum Da\int_{-b/2}^{b/2}\left((g''(y))^2-2\left(\frac{m\pi}{a}\right)^2 g(y)\,g''(y)+\left(\frac{m\pi}{a}\right)^4(g(y))^2\right)dy$$
$$-\sum q\frac{2a}{m\pi}\int_{-b/2}^{b/2} g(y)dy \quad (28)$$

The integrand in Π is:

$$F=Da\left((g''(y))^2-2\left(\frac{m\pi}{a}\right)^2 g(y)g''(y)+\left(\frac{m\pi}{a}\right)(g(y))^2\right)-\frac{2aq}{m\pi}g(y) \quad (29)$$

Or, by division by *Da*,

$$F_1^*=(g''(y))^2-2\left(\frac{m\pi}{a}\right)^2 g(y)g''(y)+\left(\frac{m\pi}{a}\right)^4(g(y))^2-\frac{2q}{m\pi D}g(y) \quad (30)$$

where F, and F_1^* are integrands of Π which depends on y, $g(y)$ and $g''(y)$

4.3 Euler Lagrange differential equation

The total potential energy functional is minimized when

$$\delta\Pi=0 \quad (31)$$

This corresponds to the Euler Lagrange differential equation:

$$\frac{\partial F}{\partial g}-\frac{d}{dy}\left(\frac{\partial F}{\partial g'}\right)+\frac{d^2}{dy^2}\left(\frac{\partial F}{\partial g''}\right)=0 \quad (32)$$

From Equation (32), the Euler-Lagrange differential equation is

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$$g^{iv}(y) - 2\left(\frac{m\pi}{a}\right)^2 g''(y) + \left(\frac{m\pi}{a}\right)^4 g(y) = \frac{q}{m\pi D} \tag{33}$$

4.4 Solution of the Euler-Lagrange equation

The Euler-Lagrange differential equation is solved using the method of trial functions, to obtain the homogeneous solution as:

$$g(y) = A_m \cosh\frac{m\pi y}{a} + B_m \frac{m\pi y}{a}\sinh\frac{m\pi y}{a} + C_m \frac{m\pi y}{a}\cosh\frac{m\pi y}{a} + D_m \sinh\frac{m\pi y}{a} \tag{34}$$

where A_m, B_m, C_m and D_m are the four constants of integration.

The particular solution $g_p(y)$ for uniformly distributed load $q(y) = q_0$ is given as

$$g_p(y) = \frac{q_0 a^4}{(m\pi)^5 D} \tag{35}$$

This problem is symmetric, hence $C_m = D_m = 0$ since the terms of $g(y)$ associated with them are odd functions of y. Thus the general solution is:

$$w(x,y) = \sum\left(c_{2m}\cosh\frac{m\pi y}{a} + c_{3m}\frac{m\pi y}{a}\sinh\frac{m\pi y}{a} + \frac{4q_0 a^4}{(m\pi)^5 D}\right)\sin\frac{m\pi x}{a} \tag{36}$$

The two constants of integration c_{2m} and c_{3m} are found using the geometric boundary conditions $w\left(x, y = \pm b/2\right) = 0$ and the force boundary conditions $\frac{\partial^2 w}{\partial y^2}\left(x, y = \pm b/2\right) = 0$

From the force boundary conditions

$$c_{2m}\cosh\frac{m\pi b}{2a} + c_{3m}\left(2\cosh\frac{m\pi b}{2a} + \frac{m\pi b}{2a}\sinh\frac{m\pi b}{2a}\right) = 0 \tag{37}$$

Hence

$$c_{2m} = -c_{3m}\left(2 + \frac{m\pi b}{2a}\tanh\frac{m\pi b}{2a}\right) \tag{38}$$

From the geometric boundary conditions,

$$c_{2m}\cosh\frac{m\pi b}{2a} + c_{3m}\left(\frac{m\pi b}{2a}\sinh\frac{m\pi b}{2a}\right) = -\frac{4q_0 a^4}{(m\pi)^5 D} \tag{39}$$

Solving simultaneously, the constants are obtained as:

$$c_{3m} = \frac{2q_0 a^4}{D(m\pi)^5\cosh\frac{m\pi b}{2a}} \tag{40}$$

$$c_{2m} = \frac{-2q_0 a^4\left(2 + \frac{m\pi b}{2a}\tanh\frac{m\pi b}{2a}\right)}{D(m\pi)^5\cosh\frac{m\pi b}{2a}} \tag{41}$$

This completely determines the deflection.

Center deflection

The maximum deflection occurs at the plate center ($x = a/2$, $y = 0$), and is given by:

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$$w_{\max} = w\left(x = {}^{a}\!/_{2},\ y = 0\right) = \sum_{m=1}^{\infty}\left(c_{2m} + \frac{4q_0a^4}{(m\pi)^5 D}\right)\sin\frac{m\pi}{2} \tag{42}$$

$$w_{\max} = \frac{4q_0a^4}{\pi^5 D}\sum_{m}^{\infty}\frac{(-1)^{\frac{m-1}{2}}}{m^5}\left\{1-\left(\frac{1+\frac{m\pi b}{4a}\tanh\frac{m\pi b}{2a}}{\cosh\frac{m\pi b}{2a}}\right)\right\} \tag{43}$$

$m = 1, 3, 5, 7, \ldots$

Bending Moment Expressions

From the bending moment displacement relations, the bending moments are obtained as:

$$M_{xx} = -D\sum\left\{-\alpha_m^2(c_{2m}\cosh\alpha_m y + \alpha_m y c_{3m}\sinh\alpha_m y + \frac{4q_0a^4}{(m\pi)^5 D}\right\}\sin\alpha_m x$$
$$+\mu\left(\alpha_m^2 c_{2m}\cosh\alpha_m y + c_{3m}(2\alpha_m^2\cosh\alpha_m y + \alpha_m^2 y\sinh\alpha_m y)\right)\sin\alpha_m x \tag{44}$$

$$M_{yy} = -D\sum\left\{c_{2m}\alpha_m^2\cosh\alpha_m y + c_{3m}(2\alpha_m^2\cosh\alpha_m y + \alpha_m^3 y\sinh\alpha_m y\right\}\sin\alpha_m x$$
$$+\mu\left(-\alpha_m^2\left(c_{2m}\cosh\alpha_m y + c_{3m}\alpha_m y\sinh\alpha_m y + \frac{4q_0a^4}{(m\pi)^5 D}\right)\right)\sin\alpha_m x \tag{45}$$

where $\alpha_m = {}^{m\pi}\!/_{a}$

Maximum bending moments occur at the plate center where $x = a/2$, $y = 0$ and are given by:

$$M_{xx} = -D\sum\left\{-\alpha_m^2\left(c_{2m} + \frac{4q_0a^4}{(m\pi)^5 D}\right)\sin\frac{m\pi}{2} + \mu\left(\alpha_m^2 c_{2m} + 2\alpha_m^2 c_{3m}\right)\sin\frac{m\pi}{2}\right\} \tag{46}$$

$$M_{yy} = -D\sum\left\{\left(\alpha_m^2 c_{2m} + 2\alpha_m^2 c_{3m}\right)\sin\frac{m\pi}{2} - \mu\alpha_m^2\left(c_{2m} + \frac{4q_0a^4}{(m\pi)^5 D}\right)\sin\frac{m\pi}{2}\right\} \tag{47}$$

For $m = 1$, $b/a = 1$

$$c_{21} = \frac{-2q_0a^4\left(2 + \frac{\pi}{2}\tanh\frac{\pi}{2}\right)}{D\pi^5\cosh\frac{\pi}{2}} \tag{48}$$

$$c_{21} = -2.742459\frac{q_0a^4}{D\pi^5} \tag{49}$$

$$c_{31} = \frac{2q_0a^4}{D\pi^5\cosh\frac{\pi}{2}} \tag{50}$$

$$c_{31} = 0.797074\frac{q_0a^4}{D\pi^5} \tag{51}$$

$$M_{xx} = 0.051668 q_0 a^2 \tag{52}$$

For $m = 1, 3$; $b/a = 1$

$$M_{xx} = 0.0472387 q_0 a^2 \quad (53)$$

For $m = 1, 3, 5$; $b/a = 1$

$$M_{xx} = 0.0479 q_0 a^2 \quad (54)$$

Similarly for $m = 1$,

$$w_c = 0.00411 \frac{q_0 a^4}{D} \quad (55)$$

For $m = 1, 3$

$$w_c = 0.00406 \frac{q_0 a^4}{D} \quad (56)$$

The expressions for the maximum deflection and the maximum bending moments are evaluated at the plate center for various values of the aspect ratio, and tabulated in Table 1.

Table.1: Coefficients for maximum deflection and maximum bending moments in simply supported plates

b/a	$w_{\max} = \alpha \frac{q_0 a^4}{D}$	$M_{xx} = \beta_{xx} q_0 a^2$	$M_{yy} = \beta_{yy} q_0 a^2$
1	0.00406	0.0477	0.0479
1.2	0.00564	0.0626	0.0501
1.4		0.0753	0.0506
1.5	0.00772	0.0812	0.0498
1.6		0.0862	0.0493
1.7	0.00883	0.0908	0.0486
1.8		0.0948	0.0479
2	0.01013	0.1017	0.0464
3	0.01223	0.1189	0.0404
4	0.01282	0.1235	0.0384

V. RESULTS AND DISCUSSIONS

The Kantorovich-Vlasov method has been successfully applied to the bending problems of simply supported rectangular Kirchhoff-Love plates subjected to uniformly distributed transverse loads over the entire plate domain. The problem of bending of Kirchhoff-Love plates under transverse distributed load was presented in variational form and the total potential energy functional for Kirchhoff-Love plates presented as Equation (2). Vlasov procedure was adopted to construct the displacement coordinate functions in the x-direction as Equation (23). Kantorovich method was adopted to consider the displacement function of the plate as the series given by Equation (24). This assumed displacement function, Equation (24) was substituted into the total potential energy functional to obtain the total potential energy functional expressed by Equation (28), where the integrand was found to depend on the functions, $g(y)$, and $g''(y)$. The Euler-Lagrange differential equation corresponding to the extremization of Π was obtained as Equation (33), a fourth order ordinary differential equation in $g(y)$ and its derivatives. The Euler-Lagrange differential equation was then solved using the trial function method to obtain the solution as Equation (36) after taking consideration of symmetry of plate and loading. Geometric (displacement) and force boundary conditions were used to obtain the constants of integration as Equations (40) and (41). The maximum deflection was found to occur at the plate center, in line with symmetry and the maximum deflection was obtained as Equation (43). Bending moment expressions obtained using the bending moment displacement relations, are given by Equations (44) and (45). Maximum bending moments were found to occur at the center of the plate, and were obtained as Equations (46) and (47).

The expression for maximum deflection, and maximum bending moments in x and y directions at the center of the plate were found to vary with *b*/*a*, the plate aspect ratio. They were also found to be rapidly convergent series, with convergence to the exact solution obtained for the maximum deflection with the use of only two terms of the series m = 1, 3, and convergence to exact solution obtained for the bending moments with the use of only three terms of the series (m = 1, 3, 5). The maximum deflections, and maximum bending moments were computed for varying aspect ratios (*b*/*a*) ranging from 1.0 to 4 using three terms of the series, and presented in Table 1. The table shows that the coefficients obtained for maximum deflection and maximum bending moments were exactly identical with the solutions obtained using the Levy single Fourier series method.

VI. CONCLUSIONS

The following conclusions can be made:

i. the Kantorovich-Vlasov method can be successfully applied to the solution of plate flexure problems for simply supported ends and static transverse loads.
ii. the solution obtained for deflection and moments is a single trigonometric series containing hyperbolic functions, which may not be so easily amenable to mathematical analysis.
iii. the solutions obtained for maximum deflection and maximum bending moment are single trigonometrical series that have rapidly convergent properties, hence convergent results to the exact solution for maximum deflection and maximum bending moment are obtained using a few terms of the series.

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