Review of Functionally Graded Structure

Rakesh Shambharkar

Abstract — In materials science functionally graded materials (FGM) are those in which the volume fractions of two or more materials are varied continuously as a function of position along certain dimension(s) of the structure to achieve a required variation. The multilayered materials are used in many structures. In conventional laminated composite structures, homogeneous elastic laminas are bonded together to obtained enhanced mechanical and thermal properties. At high temperature, such assembly creates stress concentration along the interfaces. This can lead to delamination, matrix cracks and other damage mechanisms which result from the sudden change of the mechanical properties at the interface between the layers. This can be overcome by using FGMs in which material properties varies continuously along thickness. Also the properties of all components can be fully utilized. For example, in a two-constituent FGM along thickness, one side may have high mechanical strength and the other side may have high thermal resistant property; thus, there are two aspects in one material. In FGMs, change of the material composition is continuous or layer wise. It is not made by simply “bonding individual materials.” Thus by combining appropriately several materials of different desirable properties the FGM with continuous variation of properties are produced without creating any stress concentrations at material interfaces.

Keywords— FGM, lamination, HOST, FOST, CPLT.

1. INTRODUCTION

Man made FGM was proposed in 1984 by material scientists in the Sendai of Japan; as a means of preparing thermal barrier materials (Yamanoushi, et al. 1990, Koizumi 1993). The main idea was to develop outer body of a space plane to be exposed to very high temperature environment (about 1700 °C), and at the same time it needed to be resistant to severe temperature change (about 1000 °C) between the inside and the outside. There is no single natural material endurable to such a condition. The idea was to fabricate a space plane body’s material with improved thermal resistance and mechanical properties by gradually changing material compositions through the thickness of the structure. They used ceramic for the outer surface exposed to high temperature environment and thermally conductive material, such as a metal, for the inner surface. In 1992, FGM was selected as one of the ten advanced technologies in Japan (Koizumi, 1997). Thus, FGM technology became very important and received attention of materials and mechanics communities around the world. Since then, an effort to develop high-resistant materials using FGMs has been continued. Continuous varying properties of a material are among the most important advantages of multi-functional materials. A future development in materials science lies in continuous variation of microstructure using nanotechnology, which in fact is in its initial stage. By doing so, materials can be developed with functionalities depending on specific use. In fact, functionally graded structures can be seen all around. It can be seen in nature, in bio-tissues of animals such as bones and teeth, and plants.

There are several fields which are using FGMs. These are aerospace structures, production of industrial materials such as forming tools, optoelectronics, biomaterials and energy production materials, etc. Investigations based on thermal stresses in elastic bodies are numerous. The graded transition of composition across an interface of two materials (for instance, metal and ceramics or polymer) can essentially reduce the thermal stresses and stress concentration at intersection/interfaces with free surfaces. Similarly the stress intensity factor at the crack tip can be altered by varying the gradient properties of materials across the interface. The ceramic-metal FGMs exhibit higher fracture resistance which results in higher toughness. Varying thermal expansion in graded layers induces residual stress and affects the crack growth mode. In fact, the interface bonding is much improved by providing smooth composition variation. The interest in graded materials focused primarily on the control of thermal stresses in elements exposed to high temperatures (to 1600 0C), for instance in gas turbine blades, aerospace structures, solid-oxide fuel cells, energy conversion systems using thermoelectric or thermionic materials.
Recent experimental and theoretical works demonstrated that controlling gradients in thermal and mechanical properties provides a new potential for design of surfaces and interfaces. These surfaces/interfaces endure to higher resistance to cracking and wear under the extreme mechanical surface loading and thermal gradients. The diverse applications include load-bearing engineering structures, protective coatings, bio implants and magnetic storage media.

The gradation in properties of the material reduces thermal stresses, residual stresses, and stress concentration factors. Structures made of FGM, survive a large temperature gradient in comparison to normal ones while maintaining their structural integrity. Demand of such structures is growing as it has been used in automotive, rocket industry, etc. There are several types of FGMs that exhibit exceptional multifunctional properties and multi sector applications as shown in Table 1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Properties</th>
<th>Processing</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceramic/metal bulk FGMs</td>
<td>Thermal stress relaxation; high heat resistance and wear resistance; high mechanical strength</td>
<td>Spark plasma sintering process</td>
<td>Co-injection moulding and co-sintering (building a graded interface)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High efficiency engine components</td>
<td>Automotive industry, sensors; medical instruments</td>
</tr>
<tr>
<td>Titanium (alloys) with graded density or porosity</td>
<td>Combination of good mechanical properties and light weight</td>
<td>Additive, layer-wise process: direct metal laser sintering (DMLS) of powders</td>
<td>High specific surface and strong gas-metal interaction; graded porosity combines optimised contact on substrates (bulk side) and high functional (nano-structured side)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Light weight structures for aircraft and space industry, implants</td>
<td>PVD based on sputter techniques and inert gas evaporation and condensation, with in-situ design of the deposed structures by controlling the process parameters</td>
</tr>
<tr>
<td>Tool steels with C, V, Cr gradients ; steels or Ni super alloys with ceramic (oxide, carbide) particle</td>
<td>Combination of toughness and hardness or wear resistance</td>
<td>Additive, layer-wise process: 3D-printing with local material composition control (generating a green part of powdered material and</td>
<td>Precious metals like Pt, Ag (catalysis and metal oxides like SnO₂ (sensors) with graded porosity from bulk to nanometre scale)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tools, medical instruments, implants, aircraft and space industry.</td>
<td>Magnetic and non-magnetic; ductile and stiff and other</td>
</tr>
</tbody>
</table>

Table 1 Multifunction properties and multi-sector application of FGMs
The review of literature pertinent to the subject matter is presented as:

a. Functionally Graded Materials (FGMs).
b. Review of Functionally Graded (FG) Beams and Arches.
c. Review of Functionally Graded (FG) Plates

d. Review of Functionally Graded (FG) Shells

II. FUNCTIONALLY GRADED MATERIALS (FGMS)

The new class of materials is continuously emerging, which demonstrates variation of material properties mainly in the thickness direction. Such a class of materials is called functionally graded materials (FGMs). FGMs are composite materials, microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. Various micromechanical models such as Mori-Tanaka and self consistent Schemes (Vel and Batra 2004) have been developed and employed over the years to find the effective properties of the material. But the following two mathematical representations based on volume fraction of constituent materials are used in mechanical analysis of FGM are:

Exponential Law, which is more common in fracture studies of FGM, is given as

\[ P(z) = P_0 \exp \left( -\frac{\delta}{1 - \frac{2z}{h}} \right) \]

where, \( \delta = \frac{1}{2} \log \left( \frac{P_{t}}{P_{b}} \right) \)

The power law, which is use more common in stress analysis of FGM, is given as

\[ P(z) = (P_{t} - P_{b}) \left( \frac{z}{h} + \frac{1}{2} \right)^N + P_{b} \]

where, \( P(z) \) denotes a typical material property (E, G, ρ, α).

Pt and Pb denotes the material properties at topmost and bottommost layer of the element respectively, and n is a parameter that dictates the material variation profile through the thickness. The working range of N can be taken as 1/3 to 3 depending on the application and working environments.

Considerable researches have focused on the thermoelastic, dynamic and buckling analyses of FG plates and shells in recent years, because of the varied range of advantages compared with the traditional and fibre-reinforced laminated composite materials, which are as follows:

1. FGMs can survive environments with high temperature gradients while maintaining their structural integrity.
2. Unlike the fibre-reinforced laminated composite materials having mismatches of mechanical properties across an interface due to two dissimilar layers bonded together, FGMs gradually and continuously vary the mechanical properties across the thickness direction by changing the volume fractions of two materials as desired.
3. It was reported that the weakness of the fibre-reinforced laminated composite materials such as debonding, huge residual stress, locally large plastic deformations, etc. can be avoided or reduced in FGMs.

Hence, FGMs have an enormous application potential, especially for working in the high-temperature environments. The literature is classified according to structural element beams, arches, plates and shells and it’s further classified in the refinement/accuracy of the theories.

III. FUNCTIONALLY GRADED BEAMS

FOSTs/HOSTs/HOSNTs/Exact for FG beams

Introduction to various beam theories was given by Manjunatha and Kant (1993a and 1993b) for the analysis of composites and sandwich beams. Various researchers studied the 2D beams under the 1D assumption. For FG beams, Gopalakrishnan et al. (2003) developed a new beam element for the study of thermoelastic behaviour using the first-order shear deformation theory (FOST). It has found that the presence of FGM layer in structures results in significant difference in its response from its parent material beams due to the presence of coupled stiffness and inertial parameters. Gopalakrishnan and Chakraborty (2003) also analyzed the wave propagation behavior in a FG beam subjected to high frequency impulse loading which can be either thermal or mechanical, based on the FOST. Subbaiah (2005) studied static and free vibration response of FGM beams using FOST and various HOSTs. Kapuria et al. (2006) developed formulation for coupled efficient zigzag, third order, consistent third order and first order models for hybrid piezoelectric layered FGM beams under thermo-electro-mechanical load. Aydogdu and Taskin (2007) investigated the free vibration of simply supported FG beam using different HOSTs and Classical beam theory. Kadoli et al. (2008) provided Static behavior of FG beams under ambient temperature based on HOST.

Sankar (2001) presented a 2D plane strain elastic solution for FG beams subjected to transverse loads by assuming
the young’s modulus of the beam to vary exponentially through the thickness, and the Poisson ratio was held constant. Stress concentrations were seen lesser when the softer side of FG beam is loaded and higher when the stiffer side of the beam is loaded compared to homogeneous beams. Sankar and Venkatraman (2001) further applied the above method to study the response of sandwich beam with FG core subjected to transverse mechanical loads. Their result showed that the FG core reduces the core/facet sheet interface shear stress and the normal stress and are independent of the variation in core properties. Sankar and Tzeng (2002) solved thermoelastic equilibrium equations in closed form solution to obtain the axial stress distribution. They implemented layerwise laminate theory into a linear beam element in order to provide a more accurate representation of the transverse and shear effects that are induced by increased inhomogeneity introduced through-the-thickness by using FGMs. Wu et al. (2005) presented semi-inverse method for axially FG beams with an anti-symmetric vibration mode. A study devoted to the vibration and stability behavior of FGM spinning circular cylindrical thin-walled beams was presented by Oh et al. (2005). A thermal buckling and vibration analysis of FG sandwich beam having constrained viscoelastic layer has carried out by Bhangale and Ganesan (2006) using finite element method. Low-velocity impact response developed by Apetre et al. (2006) for sandwich beams with FG core using a combination of Fourier series and Galerkin method. Ying et al. (2008) developed exact solutions for bending and free vibration of FG beams resting on a Winkler–Pasternak elastic foundation based on the two-dimensional theory of elasticity.

IV. FUNCTIONALLY GRADED ARCHES

Eslami et al. (2006) presented a mechanical buckling of curved beam made of FGMs using approximate function for displacement component. The equilibrium and stability equations of curved beams under mechanical loads are derived using the variational principle. The closed form solutions for buckling load related to the uniform compression and pure bending moment are obtained. The results presented in terms of mechanical buckling loads of curved beams made of isotropic materials for the validation. It has been concluded that the buckling load for curved FGM beam is greater than the corresponding load of isotropic metallic curved beam.

Lim et al. (2009) investigated temperature dependent in-plane vibration of FG circular arches based on 2D theory of elasticity. A new analytical solution were developed using state space formulation and Fourier series expansion obtained for simply supported circular arch. For free vibration, both the elastic properties and mass density were followed an exponential law with the same gradient index to derive an exact solution. It was concluded that the gradient index influences the frequency. The similar work was presented by Malekzadeh (2009) in the analysis of free vibration FG arches by 2D theory of elasticity. Hamilton’s principle was used to derive the equation of motion and related boundary condition. For the solution of these equations the differential quadrature method (DQM) was used. As the numerical results were not available for FGM arches, the results were validated for the isotropic material. The results give the lower natural frequency as compared to isotropic material. Malekzadeh et al. (2009) presented the in-plane free vibration analysis of the FG circular arches with temperature-dependent material properties subjected to thermal environment using FOST as displacement component. The equations of motion and the related boundary conditions subjected to initial thermal stresses were derived by using Hamilton’s Principle. These initial thermal stresses were obtained by solving the thermoelastic equilibrium equations of the arch using DQM. The methodology has the capacity to provide solutions for arches with different types of boundary conditions. The effects of the temperature rise, boundary conditions and the material graded index as well as the different geometrical parameters such as the thickness-to-mean radius ratio and the opening angle on the frequency parameters of the FG arches were investigated. It was shown that the temperature-dependence material properties can have significant impacts on the natural frequencies. Pi et al. (2010) presented a nonlinear in-plane elastic buckling analysis of circular shallow arches that were subjected both uniform temperature field and uniform radial load field. A virtual work method was used to establish nonlinear equilibrium equations and buckling equilibrium equations, and analytical solutions for the limit instability and bifurcation buckling loads were obtained. It was found that the temperature influences the limit instability, bifurcation buckling and postbuckling behaviour of shallow arches significantly. The limit instability and bifurcation buckling loads increase with an increase of the temperature. A maximum temperature is shown to exist for the occurrence of bifurcation buckling of shallow arches, and when the temperature is higher than this value, bifurcation buckling of an arch is not possible. An arch geometric parameter was introduced to define switches between the limit instability and bifurcation buckling modes, and between buckling and no buckling. It is also found that the limiting values of the
arch geometric parameter decrease with an increase of the temperature.

V. FUNCTIONALLY GRADED PLATES

Pagano (1969, 1970) has developed exact solutions of simply supported laminated plates by using 3D elasticity theory. His benchmark solutions have proved to be very useful in assessing 2D approximate plate theories (Pandya and Kant, 1988a,b; Kant and Mallikarjuna, 1991; Kant and Manjunatha, 1994; Reddy, 1996; Kant and Swaminathan, 2001, 2002). Pagano’s method is valid for laminated plates and shells, where the material properties are piecewise constant, but this method is not valid for finding solutions of plate problems with continuous inhomogeneity such as the FGMs. A lot of literature also available on study of FG plate as discussed in next section on the accuracy of the theories such as CPT, FOST, HOST and 3D elasticity theory.

VI. CLASSICAL PLATE THEORY

Exact relationships between the bending solutions of CPT and the FOST were developed for FG circular plates by Reddy (1999). Then, exact solutions of FG plates using FOST were presented in terms of the solutions of CPT for a number of boundary conditions. Finally, numerical solutions of FG plates were presented as a function of the modulus ratio and ratios of volume fractions. Ng et al. (2000) investigated the dynamic stability of FGM simply supported rectangular plates by harmonic in-plane loading using Classical Plate Theory (CPT). The results were presented for various modes and loading configurations. Yang and Shen (2001) used the formulation of CPT to investigate the dynamic response of a FG rectangular thin plate subjected to partially distributed impulsive lateral loads, with or without resting on elastic foundation. Najafizadeh and Eslami (2002) presented the formulation on CPT to study the buckling analysis of radially loaded solid circular plate made of FGM. Ma and Wang (2003) investigated axisymmetric large deflection bending and post-buckling behaviour of a FG circular plate based on CPT under mechanical, thermal and combined thermal-mechanical loadings respectively. Tsukamoto (2003) presented an analytical description of thermal-stresses in a FGM plate subjected to through-thickness heat flow by combining micromechanical with macromechanical approaches. The micromechanical approach from Eshelby’s equivalent inclusion method and Mori–Tanaka’s mean field approximation, and the macromechanical approach were based on the CPT. Zenkour (2005a, b) presented the general formulation of FG sandwich plates using sinusoidal deformation theory. Results can be obtained by assuming the strain to be zero in transverse direction. Deflection, stresses, buckling and free vibration of FG Sandwich plate investigated with sinusoidal shear deformation theory. Shariat and Eslami (2006) derived the equilibrium, stability, and compatibility equations of an imperfect FG plate using the CPT with the power law composition of the constituent materials. Ebrahimi and Rastgo (2008) developed analytical investigation of free vibration behaviour of thin circular FG plates made of piezoelectric (PZT4) material based on CPT.

VII. FIRST ORDER SHEAR DEFORMATION THEORY

The models based on the CPT are computationally more efficient but may be inaccurate in the case of thick laminates as transversal shear strains are neglected. The CPT, which totally ignores transverse shear deformation, can be better applied to the thin structures (e.g. beams, plates, and shells). To overcome this limitation of CPT, a simple shear deformation theory was developed with the transverse shear strains assumed to be constant through the thickness. Such a theory is known as First Order Shear Deformation Theory (FOST). Reissner (1945) proposed the theory assuming a stress field. These theories assumed constant shear stress/strain within each layer. A correction coefficient has to be used to adjust the transverse shear stresses. Reissner (1945) and Mindlin (1951) are the two pioneers of FOST for the analysis of the plate based on the assumed stress and the assumed displacement field, respectively. Various refined theories for composite plates were developed by Pandya and Kant, (1988a, b); Kant and Mallikarjuna (1991); Kant and Manjunatha (1994); Reddy (1996); Kant and Swaminathan (2001, 2002).

Reddy and Chin (1998) developed a finite element formulation for the dynamic thermoelastic responses of FG cylinders and plates using the FOST. Axisymmetric bending and stretching of FG solid and annular circular plates is studied by Reddy et al. (1999) using the FOST. Numerical results for displacements and stresses were presented for various percentages of ceramic-metal volume fractions. Equilibrium and stability equations of a moderately thick rectangular plate made of FG materials under thermal loads are derived by Lanhe (2004) on the basis of FOST. Buckling load of clamped FG rectangular plates resting on elastic foundations and subjected to uniform edge compression is studied by Yang et al. (2005a) using FOST. Batra and Jin (2005) investigated free vibrations of a FG anisotropic rectangular plate with help of FOST based finite element method. Thermal buckling of a simply supported FG skew plate is
investigated by Ganapatli and Prakash (2006) using FOST with finite element (FE) approach, same study adopted by Park and Kim (2006) whereas, Na and Kim (2006) investigated thermal buckling problem using FEM based on FOST. Efraim and Eisenberger (2007) presented the vibration analysis of thick annular plates with made of isotropic material and FGM (silicon nitride-stainless steel) using FOST. Fazlizadeh et al. (2007) investigated the applicability of differential quadrature method for vibration analysis of aero-thermoelastic thin-walled blades made of FG materials. FOST is used to solve the governing equations of beams which include the effects of the rotary inertias and the blade presetting angle. Fallah and Noiser (2008) gives analytical solution for non-linear cylindrical bending of FG plates subjected to thermal, mechanical, and combined thermal–mechanical loadings. The equilibrium equations within FOST and based on von Karman assumptions for large deflections are derived using variational formulations. Nguyen et al. (2008) have proposed the FOST model for FG material in which transverse shear factors is obtained through these plate models by using energy equivalence methods. Nguyen found that the shear correction factor is not the same as the one of the homogeneous FOST models; it is a function of the ratio between elastic moduli of constituents and of the distribution of materials through the models.

Sahraee (2009) presented the bending analysis of FG thick circular sector plates based on FOST. A free vibration analysis of FG plates presented by Zhao et al. (2009a) using the element-free Ritz method and FOST. Using same approach Zhao et al. (2009b) investigated the buckling behavior of FG Plates under mechanical and thermal loading. Akhavan et al. (2010) presented analytical solutions for free vibration analysis of moderately thick rectangular plates, which are composed of FG materials and supported by either Winkler or Pasternak elastic foundations. The proposed rectangular plates have two opposite edges simply-supported, while all possible combinations of free, simply-supported and clamped boundary conditions are applied to the other two edges. In order to capture fundamental frequencies of the FG rectangular plates resting on elastic foundation, the analysis procedure was based on the FOST.

VIII. HIGHER ORDER SHEAR AND SHEAR-NORMAL DEFORMATION THEORIES

Although the FOST yield acceptable solution, they do not accurately predict the characteristics of plates and shells as addressed by the 3D theory of elasticity. The FOSTs are inadequate to model the warping of transverse cross section, that is, the distribution of the deformed normal due transverse shear stresses. Higher order theories take into account the nonlinear vibration of in-plane displacement, transverse shear deformation, transverse normal strain depending on the type of model and the degrees of freedom used. This theory eliminates the shear correction coefficients. Hence, for accurate modeling and analysis of the thick shells higher order theories are necessary. This theory predicts the reliable result as compared to FOST and CLT. Aboudi et al. (1994, 1996, and 1999) developed a higher order theory (HOT) that enables thermo-inelastic analyses of FG plates with spatially varying microstructures in one, two and three orthogonal directions. Reddy (2000) presented theoretical formulation and finite element models based on third order shear deformation plate theory for static and dynamic analysis of the FGM plates. In this study, both Navier solutions for linear bending simply supported rectangular plates and finite element models for non-linear static and dynamics response were presented. Yang and Shen (2002) adopted the Galerkin approach and one dimensional differential quadrature approximation for the free vibration analysis and modal superposition for the transient response of the FGM rectangular plate using the formulation of HOST. The plate is assumed to be clamped at two edges and another may be simply supported. The plate is initially stressed with resting or without resting on elastic foundation. Javaheri and Esfami (2002) analyzed the thermal buckling problem of FG isotropic plates using HOST. The study concludes that the critical values of temperature are generally lower than the corresponding values of homogeneous plates. Qiana and Batra (2005) used a HOSNT and mesh free local Petrov–Galerkin method to analyze cantilever FG plate. Ferreira et al. (2005) have used collocation multi-quadratic radial basis functions to analyze static deformations of simply supported FG plate modelled by a HOST. With the help of third-order shear deformation plate theory Kim (2005) investigated vibration characteristics of initially stressed FG rectangular plates in thermal environment.

Nonlinear vibration and dynamic response of a FG plate with surface-bonded piezoelectric layers investigated by Huang and Shen, (2006) in thermal environments using HOST. Ferreira et al. (2007) performed the free vibration analysis of the FGM plates with use a displacement field with seven degrees of freedom by Kant and Pandya (1988b). Various FGM as well as several boundary conditions are proposed. Matsunaga (2008) used 2D HOST derived from the Hamilton’s principle to analyzed natural frequencies and buckling stresses of plates made of FGMs by taking into account the effects of transverse shear and normal deformations and rotatory inertia. Shiyyekar and Kant (2010) presented a static analysis of FG plate attached under electromechanical load. A higher order shear and normal deformation theory is used to model the elastic responses of FG plate subjected to voltages. Linear layer wise approximation of the electrostatic potential is proposed in the model. Talha and Singh (2010) presented an extensive study of the free vibration and static analysis of square and rectangular FG plates is presented, which is based on the HOST with a special modification in the transverse displacement in conjunction with finite element models using thirteen degree of freedom per node. The systems of algebraic equations are derived using variational approach for the free vibration and static problem. The mechanical properties of the FG plate are assumed to vary continuously in the thickness direction by a simple power-law distribution in terms of the volume fractions of the constituents.

IX. THREE-DIMENSIONAL ELASTICITY THEORY MANUFACTURING

Mian and Spencer (1998) established a class of exact three-dimensional (3D) solutions for FG plates with traction-free surfaces. The 3D thermoelastic deformations of FG isotropic elliptic plates were investigated by Cheng and Batra, (2000). It was concluded that the distributions of deformation and stress through the thickness direction are agreed for the homogeneous plate with results of Reddy (1984) but it is not the case with FG plates whose deformations depend on the volume fraction of the constituent materials. The gradients in material properties significantly affect the response of a FG plate under thermal loads. 3D solutions for the static and dynamic problems of (smart) FG plates were presented by Reddy and Cheng (2001a, 2001b, 2003). Vel and Batra (2004) presented a 3D analytical solution for free and forced vibrations of simply supported FGM rectangular plates. Numerical results for the natural frequencies and dynamic responses of circular plates are presented by the Nie and Zhong (2007) based on 3D elasticity theory. Huang et al. (2008) developed the benchmark (exact) solutions for FG thick plates resting on Winkler–Pasternak elastic foundations with the help of 3D of elasticity. Elastic deformation of coated plated subjected to transverse loading is investigated by Kashlaltian and Menshykova (2009) using 3D elasticity theory. An exact solution is presented by Pan and Han (2005) for the multilayered rectangular plate made of FG anisotropic, and linear magneto-electro-elastic materials using pseudo-Stroh formalism. Li et al. (2008) also presented the same problem for uniform load. Vel and Batra (2003) also developed an analytical solution for 3D thermo-mechanical deformations of a simply supported FG rectangular plate subjected to time-dependent thermal loads. 3D elasticity solution for bending of FG rectangular plates is given by Kashlaltian (2004). Malekzadeh (2009) developed 3D free vibration analyses of FG plates. Ootao and Tanigawa (2005) developed same for transient thermal stresses of FG rectangular plate.

Li et al. (2008b) studied free vibration of FG sandwich rectangular plates with simply supported and clamped edges, and the formulation is based on the 3D elasticity theory. Xu and Zhou (2009) studied the 3D elasticity solutions of FG rectangular plates with internal elastic line supports. The exact solutions for the displacements and stresses, which exactly satisfy the governing differential equations and the simply supported boundary conditions at four edges of the plate obtained. Convergence and comparison studies show the correctness and effectiveness of the method. Reddy and Cheng (2003) studied the harmonic vibration problem of FG plates by means of a 3D asymptotic theory formulated in terms of transfer matrix. Instead of using multiple time scales expansion, the frequency is determined in a much simpler way that renders the asymptotic method to be practically validated for finding any higher-order solutions. This is illustrated by applying the refined formulation to a FG rectangular plate with simply supported edges.

Li et al. (2011) carried out the 3D analysis of a transversely isotropic FG piezoelectric circular plate under tension and bending. They provide the direct displacement method, with analytical solutions obtained for plate with either free or simply-supported edge conditions. The material properties of the plate vary arbitrarily along the thickness. Shao (2005). Shao and Wang (2006, and 2007) studied the thermo-mechanical stresses of FG hollow cylinders and cylindrical panels. An approximate static solution of FG hollow cylinders with finite length was obtained by using of multi-layered method; analytical solution of FG cylindrical panel was
carried out by using the Frobenius method and analytical solution of transient thermo-mechanical stresses of FG hollow cylinders were derived by using the Laplace transform technique and the power series method, in which effects of material gradient and heat transfer coefficient on time-dependent thermal mechanical stresses were discussed in detail.

X. FUNCTIONALLY GRADED SHELL

Classical Theory

Loy et al. [1999] investigated the free vibration of simply supported FG cylindrical shells made up of stainless steel and nickel. This investigation was later extended by Pradhan et al. (2000) presented the free vibration analysis of FG isotropic cylindrical shells consisting of stainless and zirconia. Effects of boundary conditions and volume fractions on the natural frequencies of FG cylindrical shells were studied. Nonlinear post buckling analysis of FG cylindrical shells was presented by Shen (2002) based on the classical shell theory with von-Karman–Donnell type of kinematic non-linearity. Similarly, based on the same theory, Shen (2003) studied on the postbuckling of pressure-loaded FGM cylindrical shells under constant thermal load. Navazi and Haddadpour (2007) analytically investigated the aero-thermoelastic stability margins of FGM panels in thermal environment using classical approach. The loading is formulated using the quasi-steady supersonic piston theory.

Many of the classical theories were developed for thin shells and are based on Love-Kirchhoff assumptions and surveys of such classical shell theories as seen in the works of Toorani and Lakis (2000).

First Order Shear Deformation Theory

Reddy and Chin (1998) investigated the dynamic thermo-elastic response of FG cylinders using FOST. Shahsiah and Eslami (2003) presented a buckling analysis for simply supported FGM cylindrical shells under two cases of thermal loading, i.e. uniform temperature rise and linear and nonlinear gradient through the thickness, based on the FOST. In their analysis, the material properties were considered to be independent of temperature. Kadoli and Ganesan (2006) presented the displacement field based on FOST along with Fourier series expansion in circumferential direction is used to investigate the linear thermal buckling and free vibration of clamped-clamped boundary conditions of FG cylindrical shell. They have considered the high inner-temperature and ambient at outer temperature using the one-dimensional heat conduction equation along the thickness of the shell to determine the temperature variation. Liew et al. (2006) presented the linear and nonlinear vibration analysis for three layer coated cylindrical panel. Theoretical formulation is based on geometric non linearity in von karman sense and FOST which accounts for the transverse shear strain and rotary inertia. Using same formulation Ganpathi (2007) investigated the spherical shell and presented dynamic axisymmetric stability behaviour of FG spherical shell subjected to externally applied pressure. Tornabene and viola (2009) studied the free vibration of FG parabolic, circular panels and revolution of shells using the FOST. Generalized differential quadrature numerical technique is used for obtaining the eigen values. Shahsiah et al. (2006, 2010) studied thermal instability of shallow and deep spherical shells made of FG material. They used Sanders nonlinear strain displacement relation and FOST to derive the governing equilibrium and stability equation for FG cylindrical shells. It is assumed that the mechanical properties are linear functions of thickness coordinate. The constituent material of the FG shell is assumed to be a mixture of ceramic and metal. The analytical solutions are obtained for three types of thermal loadings including the uniform temperature rise (UTR), the linear radial temperature (LRT), and the nonlinear radial temperature (NRT). Sepiani et al. (2010) investigated the free vibration and buckling of a two-layered cylindrical shell structure made of elastic embedded FG shell subjected to combined static and periodic axial forces using two different methods such as the FOST considering transverse shear strains and rotary inertias, and the CST.

Higher Order Shear Deformation Theories

Gong et al. (1999) presented an elastic response analysis of simply supported FGM cylindrical shells under low-velocity impact using a third order shear deformation theory which was developed by Reddy and Liu (1985) in cylindrical co-ordinate. Wu et al. (2002) presented a higher order theory to examine the electromechanical behaviour of piezoelectric generic shells with graded material properties in the thickness direction. The material constants were power functions in the radial direction. Postbuckling analysis of FGM cylindrical shells under combined axial and radial loads in thermal environments was presented by Shen and Noda (2005). 1D steady state heat conduction through the thickness was assumed. The formulations were based on Reddy (1985) HOST. The results states that the temperature field and volume fraction distribution have a significant effect on the postbuckling behavior, but they have a small effect on the imperfection sensitivity of the FG shell. Patel et al. (2005) analyzed the free vibration characteristics of FG
elliptical cylindrical shells using finite element formulation based on HOST through the thickness approximations of both in-plane and transverse displacements. Shariyat (2008) presented dynamic buckling of a prestressed, suddenly heated imperfect FGM cylindrical shell and dynamic buckling of a mechanically loaded imperfect FGM cylindrical shell in thermal environment, with temperature dependent material properties. In contrast to all shell theories used so far, the shell theory presented by Shariyat and Esfahani (2000) was used. The finite element method was used to solve governing equation. Shen (2009) studied the postbuckling response of a shear deformable FG cylindrical shell of finite length embedded in a large outer elastic medium and subjected to axial compressive loads in thermal environments. The surrounding elastic medium is modeled as a tensionless Pasternak foundation that reacts in compression only. The postbuckling analysis is based on a HOST with von Karman–Donnell-type of kinematic nonlinearity. The thermal effects due to heat conduction are also included and the material properties of FGMs are assumed to be temperature-dependent. Pradyumna et al. (2010) carried out the investigation on the nonlinear transient analysis of FG curved panels using HOST developed by Kant and Khare (1997) for 9 DOF and also they include the twist curvature while employing the finite element method. They neglected the heat conduction between ceramic and metal constituents. Material properties are assumed to be temperature-independent and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. Pradyumna and Bandyopadhyay (2010) extended the study for dynamic instability behavior of different type of shell subjected to in-plane periodic and temperature field. They assumed that the properties of FG materials are temperature dependent and graded in the thickness direction according to the power-law distribution in terms of volume fraction of the constituents.

Three-Dimensional Elasticity Theory

Extensive investigations on the thermal bending and post-buckling of isotropic and composite plates and shells were carried out by Tauchert and Huang (1987); Tauchert (1991); Meyers and Hyer (1991) and Leissa (1992), etc. Wu and his colleagues also have established a number of 3D solutions of thermoelastic, dynamic and buckling problems of laminated composite doubly curved and conical shells (Wu et al., 1996a, 1996b; Wu and Chiu, 2001, 2002) using the method of asymptotic expansion. Other 3D linear analyses of laminated composite plates and shells can also be found in the literature (Fan and Zhang, 1992; Bhimaraddi, 1993). Wu and Tsai (2004) presented the static problem of FG annular spherical shells by combining the method of differential quadrature (DQ) with the asymptotic expansion approach in 3D elasticity solution. Na and Kim (2004) performed 3-D thermal buckling analysis of FGMs, the finite element model was adopted using an 18-node solid element to analyse more accurately the variation of material properties and temperature field in the thickness direction. Santos et al. (2008) developed a semi-analytical axi-symmetric finite element model using the 3D linear elasticity theory. To study of thermoelastic analysis of FG cylindrical shells subjected to transient thermal shock loading. Vel (2010) presented an exact elasticity solution for the free and forced vibration of FG cylindrical shells. The natural frequencies and mode shapes are presented for different geometric configurations and material composition profiles.

XI. CONCLUSION (SUMMARY OF REVIEW)

The literature reviews of various approaches, such as classical, FOST, HOST and 3D topics were presented in preceding sections. For study of static, free and forced vibration, stability, thermal loading, thermal-buckling, postbuckling analysis of FG arches, beams, plates and shell were presented. Few general remarks relating to the literature are listed below.

- Elasticity solutions predicts the displacements and stresses accurately than the classical, FOSTs, and HOSTs, but their solution methods involve mathematical complexities and are very difficult and tedious to solve.
- The 2-D in plane elements based on the classical approach is suitable for thin elements, but it under-predicts the deflections and over predicts the natural frequencies and buckling loads in relatively thick elements.
- The models based on the FOST are very often used owing to their simplicity in analysis and programming. However, it requires a convenient value of shear correction coefficient. In practice, this coefficient has been assumed to be 5/6 for homogeneous materials. This value is no longer appropriate for FGM analysis due to the position dependence of elastic properties.
- The model based on HOST considers the transverse shear strain and normal strain through the thickness of the element. The theory does not required shear correction coefficients.
REFERENCES


[46] Lanhe, W., (2004), Thermal buckling of a simply supported moderately thick rectangular FGM plate, Composite Structures, 64, 211–218.


[58] Matsunaga, H., (2008), Free vibration and stability of functionally graded plates according to a 2-D higher-order deformation theory, Composite Structure, 82, 499–512.


[75] Pan, E. and Han, F., (2005), Exact solution for functionally graded and layered magneto-electro-


[100] Sahraee, S., (2009), Bending analysis of functionally graded sectorial plates using Levinson plate theory, Composite Structures, 88, 548-557


